Performance Of Global Geopotential Earth Models Over Egypt As An Alternative For The Unavailable Accurate Local Geoid Model

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ملخص البحث:

عكف حديث بعض الجهات البحثية مثل وكاله الخرائط العسكرية الأمريكية (DMA) و مركز جودارد الفضائي التابع لوكالة الفضاء الأمريكية (NASA) علي إنشاء نماذج جيويد عالية. لذا فقد وجد أنه من الضروري أن نعمل علي تقييم نماذج الجيويد العالمية المتاحة وان نحاول تعيين أفضل هذه النماذج مناسبة للقطر المصري. وقد بني التقييم علي مقارنة القيم المحسوبة من هذه النماذج بالقيم المناظرة المحسوبة من الأرصاد الأرضية وأرصاد الأقمار الصناعية وذلك لمناطق متفرقة بالقطر المصري. ولذلك تم استخدام نوعان من البيانات في إجراء الاختبارات اللازمة في هذا البحث. النوع الأول عبارة عن أرصاد GPS/Leveling تمت بواسطة كلاً من المشروع الفنلندي (FINNMAP-89) والشبكة القومية الجيوديسية عالية الدقة لهيئة المساحة المصرية (HARN-96) و شبكة المطارات المصرية (CAA-98) أما النوع الثاني من البيانات المستخدمة فيتكون من شبكة مربعة (5'x5') لقيم حيود الجيويد المستنبطة من نماذج الجيويد العالمية المتاحة وهي: (OSU81, Geoid-84, OSU86, OSU89, OSU91, OSU91A and EGM96) وقد أظهرت النتائج عدم وجود فروق ملموسة بين نماذج الجيويد العالمية المختبرة للقطر المصري وبعضها البعض. وقد وجد أيضا أنه بالتعامل مع القطر المصري كله كوحدة واحدة أن النموذج العالمي EGM96 قد أعطي نتائج أفضل قليلا من بقية النماذج الأخرى وذلك لعملية تمثيل قيم حيود الجيويد. و مع هذا فانه بالتعامل مع القطر المصري في صورة مناطق مختلفة طبقاً لطبيعة البيانات المتاحة تبين أنه لا يوجد نموذج بعينه يناسب كل المناطق معا. وعلي هذا فقد روعي في هذا البحث تعين النموذج الأكثر مناسبة لكل منطقة درست من هذه المناطق.

Abstract

Recently, several new accurate global geopotential earth models have been developed by several organizations, such as: U.S. Defense Mapping Agency (DMA) and NASA Goddard Space Flight Center (GSFC). Such global geopotential models have been proven to be very efficient, as alternative to the unreliable local or regional models, over areas lacking required geodetic and gravimetric data. Therefore, it is crucial to evaluate the available global geopotential earth models and to determine the most suitable one for the Egyptian territory. Such an evaluation can be undertaken by comparing quantities computed by these models with surface and satellite data in different regions allover Egypt. In this research, the adopted strategy is to compare geoid models derived from geopotential coefficients against GPS ellipsoidal heights located on optically leveled stations.

Two types of data are utilized in the numerical tests. First, GPS/Leveling data available from each of the FINNMAP Project 1989, the Egyptian National Geodetic Network HARN-1996, and the Egyptian Airports GPS Network CAA-1998. Second, a 5' x 5' gridded mean geoidal undulations derived from the available geopotential models; OSU81, Geoid-84, OSU86, OSU89, OSU91, OSU91A and EGM96. The results showed that there are insignificant differences between the tested geopotential models. Treating the whole country as one unit, the geopotential model EGM96 has given slightly better results than the other tested models. However, when splitting the country into separate regions, there is no particular geopotential model that suits all regions together. An attempt is carried out to assign the best suitable geopotential model for each studied region.

1. Introduction

There are many circumstances that affect the computation process of geoid determination. A lot of geoids allover the world have been developed in the last few decades, in consideration of the effects of many items such as the terrain effect, the density and distribution of the data, and the integration of different data sources. It is worthwhile to summarize first the results of preliminary geoid studies in Egypt. The studies have been carried out over the last few years by Al-Naggar [1986]; Nassar [1987a, 1987b]; El-Tokhey [1993]; Nassar, et al. [1993]; El-Sagheer [1995]; Shaker, et al. [1997]; Dawod [1998]; and Nassar, et al. [2000b]. Various techniques of geoid determination, according to the available geodetic data, have been considered in performing such studies. Table (1) presents a summary of the statistics of the recently derived local geoid solutions for Egypt.

Due to the lack of geodetic data over the whole globe, practicing geodesists have oriented their investigation to work with global geoidal models, known as geopotential earth models (GEMs), instead of working with unreliable regional or local geoidal models. Global geopotential models have been modeled over the past two decades to approximate the earth's gravity field on a global level. OSU81, Geoid-84, OSU86, OSU89, OSU91, OSU91A and EGM96 are some GEMs, which float around nowadays in practice. The main objective of such global models is to use them for geodetic applications over the regions lacking the availability of gravemetric quantities, particularly the gravity anomaly and geoid information. A global model is based on an exponential series of spherical harmonics that is expanded with a certain degree and order. The degree and order of any developed global geopotential model depend mainly upon the available gravimetric and other related geodetic data allover the world as well as the capacity of the available digital computers. The later factor does not yield any problems nowadays, since the high technology makes them quite sufficient for any involved capacity [Nassar, et. al., 2000a].

The present research aims at evaluating the available global geopotential earth models and determining the most appropriate geopotential model allover the entire country. One approach is to compare quantities inferred from such models with their corresponding values obtained using terrestrial as well as artificial satellite methods in different regions allover Egypt.

2. Geoid Determination Based On Artificial Satellite Techniques

The geoid determination means the computation of the geoid undulations N and/or deflection components ξ and η, at each point of interest on the terrain. In this context, different techniques for geoid determination can be used according to the available type of geodetic data in the region or country of interest. In geodetic practice, some techniques of geoid determination can be derived on the basis of using only one type of geodetic data. Some other techniques can be formulated by combining separate pairs of those data. Moreover, geoid determination techniques can be further expanded to encompass all types of available geodetic heterogeneous data. In such a case, the least-squares collocation methodology will probably be the only possible solution As a matter of fact, geoid determination techniques are applied on local, regional or global basis. However, some techniques can provide geoidal information coverage for more than one level of those three categories [Nassar, 1987b]. Details concerning the different techniques of geoid determination are explained in many literatures. However, the details of the two main artificial satellite techniques will be summarized in the following subsections.

2.1 Geometric Satellite Technique, GPS/Leveling Geoid

The GPS technique is the most recent and accurate technique for the determination of the three dimensional geodetic coordinates (ϕ, λ, h) of desired terrain points. In the geometric satellite technique, the geoid-ellipsoid linear separation, which is known as geoidal height or geoid undulations N, is to be determined at each terrain point, which will be a GPS station. The computation of this separation is accomplished by comparing the ellipsoid height h of the point of interest and the orthometric height H of the same point as follows [Mortiz, 1980]:

$$N = h - H \tag{1}$$

The computed value of geoid undulation would be absolute when the reference ellipsoid is a global geocentric one such as WGS84. It would be a relative value in the case that the reference ellipsoid is the adopted regional or local datum of the country. The deflection of the vertical components, describing the angular relationship between the geoid and the reference ellipsoid, can be determined from the geoid undulation as the slope of the geoid surface in any desired direction [Nassar, 1987a].

The geoid resulting form this technique will be of a consistent quality and can be produced more rapid than any of the conventional terrestrial techniques. Moreover, it is easy to make the satellite geoid more detailed by increasing the number of GPS stations. However, the expenses of the method are among the main factors to be considered. In regard to the accuracy of the derived geoid by this technique, it is obvious that it relies on the accuracy of each of the ellipsoidal height and the orthometric height.

2.2 Satellite Dynamics Technique

Concerning the geodetic data coverage, there are still plenty of gaps over several parts in most countries. Hence, all types of available geodetic data for geoid determination, namely: terrestrial gravity and astronomic data, satellite generated and dynamic data, and satellite altimetric data, are best combined together into a unified least squares solution, for harmonic coefficient estimation. Normally, the obtained accuracy of the currently available GEMs, for computing the geoid undulation, was estimated to be about 1-2 meter in case of GEMs with degree 180, and about 0.5 meter for GEMs with degree 360. It may be worthwhile to state here that the derived expression of spherical harmonics expansions, say to degree 360, is performed originally to the anomalous potential. However, there are well known expressions relating such anomalous potential T with all other parameters describing the anomalous gravity field, such as gravity anomaly Δg, normal gravity γ, geoid undulation N, and ... etc. [Mogahed, 1999].

The disturbing potential T can be expressed as follows [Pearse and Kearsley, 1997]:

$$T(r, \theta, \lambda) = (GM/r) \sum_{n=2}^{N_{max}} (a/r)^n \sum_{m=0}^{n} (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\cos \theta) \qquad (2)$$

where:

r	geocentric radius distance,
θ, λ	colatitude and longitude respectively,
GM	gravitational constant,
a	semi-major axis,
$\bar{C}_{nm}$ & $\bar{S}_{nm}$	fully normalized geopotential coefficients, and
$\bar{P}_{nm}(\cos \theta)$	fully normalized associated Legendre functions.

If the geopotential coefficients are given, the geoid undulations can then be determined using Brun's formula [Moritz, 1980]:

$$N = T / \gamma \qquad (3)$$

where γ is the normal gravity at the computation point.

By combining equations (2) and (3) the following equation results:

$$N(r, \theta, \lambda) = (GM/\gamma\, r) \sum_{n=2}^{N_{max}} (a/r)^n \sum_{m=0}^{n} (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\cos \theta) \qquad (4)$$

The main factor affecting the accuracy of the dynamic satellite geoid is the accuracy of the harmonic coefficients. The errors in the harmonic coefficients are due to observational errors, computational errors and truncation errors. Applying error propagation to the dynamic satellite model, the accuracy of the resulting geoid reaches

0.5 m. Studies have indicated that although the geometric satellite geoid is more accurate than the dynamic satellite geoid, the latter is less expensive [ElSagheer, 1995].

3. Methodology of Investigation

From observed GPS ellipsoidal heights h_{GPS} and leveled heights H, observed "GPS" geoidal heights are obtained by:

$$N_{GPS} = h_{GPS} - H \tag{5}$$

Geoidal heights are then compared to the geoidal heights N_{GM} derived from the used geopotential models. The difference of geoidal undulations ΔN is resulted as:

$$\Delta N = N_{GPS} - N_{GM} \tag{6}$$

A two dimensional algebraic polynomial P_k of the k-th order is usually used to express geoid undulation differences as a two dimensional surface [Vanicek and Merry, 1973]:

$$\Delta N(\phi,\lambda) = P_k(\phi,\lambda) = \sum_{i=0}^{k}\sum_{j=0}^{i} C_{ij}\phi^{j}\lambda^{i-j} \tag{7}$$

Here, ϕ and λ are local coordinates referring to the latitude and longitude of the point at which the undulation difference is required to be computed. The coefficients C_{ij} are obtained using least-squares regression. To arrive at a reliable estimate for the coefficients of the above polynomial, the knowledge of geoid undulations is required at a sufficient number of points within the area of interest. These undulations could be computed at well-distributed discrete locations within the area using the technique of geoidal surface.

To analyze the slope and bias between geoid undulations differences ΔN, the following linear fitting formula, which can be considered as a special case of equation (7), can be used [Forsberg, 1997]:

$$\Delta N = \alpha + \beta\,\phi + \gamma\,\lambda \tag{8}$$

where:

ϕ , λ	the latitude and longitude, respectively,
α	the absolute term in first-order polynomial coefficients,
β and γ	the slope in latitude and longitude, respectively.

4. Used Data Sets

A number of national GPS/leveling surveys have been gathered and utilized in this research. Figure (1) shows the location of the GPS/leveling stations that are observed in these surveys. The major GPS/leveling surveys, which have been used here, are:

a. GPS/Leveling measurements available for 387 stations in the Egyptian Eastern Desert from the FINNMAP Project 1989. The network stations are based on 56 benchmarks and are observed in 1987 using single frequency GPS receivers.

b. GPS/Leveling measurements available for 17 stations from the Egyptian National Geodetic Network, Egyptian Survey Authority HARN-1996. The network stations are observed using dual frequency GPS receivers and adjusted as one network. It is worthwhile to mention that the network consists of 30 stations. However, only 17 stations have observed orthometric heights. The other 13 stations have orthometric heights derived from the OSU91A geopotential model.

c. GPS/Leveling measurements available for 87 stations from the Egyptian Airports GPS Network, Civil Aviation Authority CAA-1998. The network stations observed from 1996-1998 using dual frequency GPS receivers and adjusted as region-by-region networks. There are seventeen local networks; each is located in one airport area. Each network consists of a number of stations that range from four to eight.

In each of the three GPS/leveling data sets, the elevations refer to the mean sea level and have the accuracy of the first and second order leveling networks. Moreover, the GPS positioning accuracy as well as the leveling accuracy at observed stations are on the few centimeters level. It can also be noted that the data sets are neither dense enough nor well distributed allover the whole country.

The computations of spherical harmonic geoid undulations were carried out in 5'x 5' grid derived from the available geopotential models. A number of spherical harmonic models have been tested; OSU81, Geoid-84, OSU86, OSU89, OSU91, OSU91A and EGM96. The statistics of geoid undulations as derived from the available investigated global geopotential earth models are listed in Table (2).

5. Performance Approach and Analysis of Results

For each of the GPS/leveling stations mentioned in section (4) and shown in figure (1), the orthometric height (H) is subtracted from the ellipsoidal height (h) to obtain the geometric geoid height ($N_{GPS} = h - H$). Then, the global geopotential model geoid height (N_{GM}) is calculated at each station using certain software, for each one of the tested geopotential models. The difference between the geoid heights at each station was then calculated ($\Delta N = N_{GPS} - N_{GM}$) and analyzed statistically. Below are the obtained results as well as their analysis:

1- The statistics of the geoid undulations derived from all the investigated global geopotential earth models are given in Table (2). One can see that, there are no significant differences to classify one of the models as much better than any other existing model in fitting the Egyptian territory.

2- The statistics of the differences between the point geoid undulations derived from each of the three GPS/Leveling data sets and those obtained from the available global geopotential earth models are listed in Tables (3), (4) and (5), respectively.

3- From Table (3) it can be noted that the mean values of ΔN are considerably large whereas the corresponding RMS values for single undulation difference determination are small. This may indicate a bias in the undulations derived from the first GPS/Leveling data set (FINNMAP-89), in the order of 10 meters.

4- The situation is different for the results given in Table (4) as computed based on the undulation obtained from the second GPS/Leveling data set (HARN-96), and also for the results given in Table (5) as based on the computed undulation from the third data set (CAA-98). In both second and third data sets, the mean undulation differences ΔN is much closer to the theoretical zero value, which indicates that both second and third data sets have much more better quality than the first data set (FINNMAP-89).

5- The results listed in Tables (4) and (5) indicate more compatibility of the third GPS/Leveling data set (CAA-98) with the tested geopotential models, than of the second GPS/Leveling data sets (HARN-96).

6- Table (6) displays the statistics of the resulted undulation differences, as obtained from using the third data set (CAA-98), after removing the small existing bias, through the fitting technique expressed by equation (8). As shown in Table (6), RMS values of geoid undulation differences ΔN get better after linear fitting with the first order polynomial. Moreover, the geopotential model EGM96 has given slightly better results than the other tested models.

7- Comparisons have been made for the geoid undulations obtained from GPS/Leveling points and geopotential models in the seventeen local GPS networks of the third data set (CAA-98). Table (7) includes the statistics of ΔN results for each region. It is shown that there is no particular geopotential model that suits all regions together. In an attempt to assign the best suitable geopotential model for each studied region, it is found that:
   a- OSU89 is the best for Cairo, Embaba, Port Said, and Assuit.
   b- EGM96 suits Alexandria, El-Arish, El-Gora, and Hurghada.
   c- Geoid-84 suits New Valley, El-Tor, Sant Katrin, Sharm EL-Sheikh and, Luxor.
   d- OSU91 fits Abou-Simbul and Marsa Matrouh.
   e- OSU81 fits Ras El-Nakab.

6. Conclusions

An attempt has been taken in the present investigation, to evaluate the available global geopotential earth models and to determine the most suitable model for the Egyptian territory. Here, comparisons are made among measured geoid undulations, utilizing three GPS/leveling data sets (FINNMAP-89, HARN-96, and CAA-98), with those computed from the available geopotential models (OSU81, Geoid-84, OSU86, OSU89, OSU91, OSU91A and EGM96). According to the obtained results, the following conclusions can be drawn:

- There are slight differences in the performance of the tested geopotential earth model over the Egyptian territory. Thus, any of the available geopotential earth models can be utilized for the representation of Egyptian local geoid. However, EGM96 has given slightly better results than the other tested models.

- In most of the cases in practice, one usually working in a certain limited local areas. In which case, the EGM96 global geopotential model has been found to be the best model to use over individual local areas. However, it was found that a different global geopotential model will fit a certain local area much better than other models. This was evident from the analysis of the obtained results made over seventeen local areas of the Egyptian airports available data set.

- The proposed fitting technique of first order polynomial was found to be practically efficient in removing the existing biases in any GPS/leveling data set. In such a case it can be used to derive more accurate geoid undulation difference between the adopted global geopotential model and GPS/leveling derived undulations more precisely, using the derived polynomial coefficients, and the geographic coordinates of the terrain points under consideration.

Based on the above conclusions, the orthometric heights could be generated from ellipsoidal heights derived from GPS measurements by using the best fitting polynomial for the undulation differences computed for each of the tested geopotential models, over each individual areas of Egypt, using the derived polynomial coefficients for the seventeen different local area. The estimated accuracy of derived orthometric heights in this case expected to be better than 2 meters. This height accuracy is sufficient for the production of topographic maps in scale of 1:10,000, which are generally sufficient for planning and management of engineering projects.

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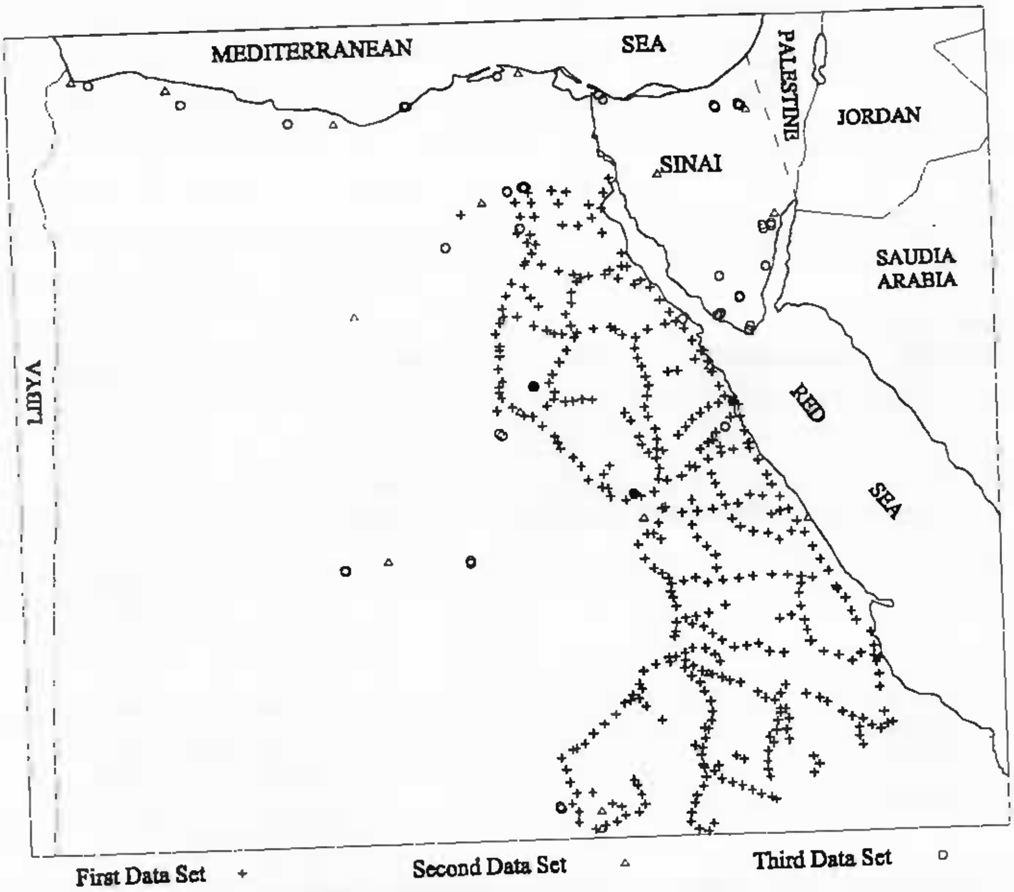

Figure (1) : Distrubition of The Available Data Sets Over Egyptian Territory.

Table (1): Statistics of Geoid Undulations of the Recently Derived Local Geoid Solutions for Egypt, in Meters.

Geoid derived By	Used Geodetic Data	Minimum Undulation	Maximum Undulation	Mean Undulation	RMS
Alnaggar, 1986	Heterogeneous	7.47	22.32	16.47	3.30
El Tokhy, 1993	Heterogeneous	13.03	35.00	23.14	5.21
El Sagheer, 1995	Gravimetric + DTM	16.87	31.32	23.19	3.71
Shaker, et al., 1997	Gravimetric + GPS	12.35	34.22	23.47	4.47
SRI-GEOID, 1998	Gravimetric + GPS	7.22	22.55	15.31	3.10
Nassar, et al., ASU-2000	Heterogeneous	7.70	22.66	14.64	0.343

Table (2): Statistics of the 5'x 5' Gridded Geoidal Undulations Derived from the Available Geopotential Models Over the Egyptian Territory, in Meters.

Geopotenial Model	N minimum	N maximum	N mean	RMS
OSU81	7.7700	23.4700	14.6780	2.9902
Geoid84	5.9593	26.6196	14.9828	4.3905
OSU86	7.8200	24.5400	15.4550	3.4438
OSU89	5.8700	23.0500	14.7136	3.5977
OSU91	7.1100	22.5800	14.8753	3.2552
OSU91A	8.6788	22.6606	14.8497	3.0759
EGM96	8.4700	22.0800	14.7422	2.6907

Table (3): Statistics of the Differences (ΔN) Among Geoidal Undulations Derived from the Geopotential Models with Those Obtained from First GPS/Leveling Data Set (FINNMAP-89), in Meters.

Geopotenial Model	ΔN minimum	ΔN maximum	ΔN mean	RMS
OSU81	7.4772	15.9649	10.8343	1.4329
Geoid84	8.3531	16.1959	12.6813	1.5102
OSU86	7.2403	15.5291	10.3644	1.5626
OSU89	8.4267	18.5495	12.7725	1.7092
OSU91	8.0003	17.4962	11.9588	1.6619
OSU91A	8.3207	17.6841	12.1224	1.8348
EGM96	7.2039	15.7925	11.2124	1.2164

Table (4): Statistics of the Differences (ΔN) Among Geoidal Undulations Derived from the Geopotential Models with Those Obtained from Second GPS/Leveling Data Set (HARN-96), in Meters.

Geopotenial Model	ΔN minimum	ΔN maximum	ΔN mean	RMS
OSU81	-7.0859	3.7402	-1.3395	4.0681
Geoid84	-9.4414	7.4421	-2.1974	5.7061
OSU86	-9.1083	2.4565	-2.9718	4.1983
OSU89	-9.0732	7.3828	-1.2429	5.9464
OSU91	-8.3374	6.7181	-1.3369	5.3659
OSU91A	-8.2514	7.0404	-1.4679	5.1510
EGM96	-6.9067	5.8186	-0.7943	4.5713

Table (5): Statistics of the Differences (ΔN) Among Geoidal Undulations Derived from the Geopotential Models with Those Obtained from Third GPS/Leveling Data Set (CAA-98), in Meters.

Geopotenial Model	ΔN minimum	ΔN maximum	ΔN mean	RMS
OSU81	-3.2198	4.5777	-0.7605	2.2888
Geoid84	-6.0519	1.3616	-2.1032	2.1811
OSU86	-4.2355	5.6288	-1.4993	2.9812
OSU89	-3.1167	7.4007	1.0521	3.6273
OSU91	-3.5033	6.2320	0.2848	3.3925
OSU91A	-3.3519	3.5907	-0.5449	2.3486
EGM96	-2.1938	4.2541	0.1183	2.1031

Table (6): Statistics of the Differences (ΔN) Among Geoidal Undulations Derived from the Geopotential Models with Those Obtained from Third GPS/Leveling Data Set (CAA-98), in Meters, After Linear Fitting.

GM	Polynomial Coefficients			ΔNmin.	ΔNmax.	ΔNmean	RMS
	Abs. T.	ϕ – Dir.	λ – Dir.				
OSU81	-1.4011	-0.407127	0.392851	-2.8433	0.7232	-0.6427	0.9650
Geoid84	4.1018	-1.015650	0.738017	-5.4743	1.0798	-1.9961	1.9888
OSU86	-18.8852	-0.554367	1.039050	-5.0617	1.3563	-1.3858	1.9080
OSU89	-18.7513	-0.923093	1.448200	-4.1759	5.2501	1.2158	2.7744
OSU91	-15.0890	-0.962664	1.351800	-4.9460	4.2073	0.2848	2.6875
OSU91A	-9.6869	-0.781429	0.987369	-4.4922	2.4203	-0.5449	2.0316
EGM96	-11.8382	-0.556856	0.870902	-3.1272	2.5469	0.1183	1.6699

Table (7): Statistics of the differences (ΔN) Among Geoidal Undulations Derived from the Geopotential Models with Those Obtained from GPS/Leveling Data of Each Individual Airport area, in Meters, After Linear Fitting.

Zone	St. No.	Used GM	Polynomial Coefficients			ΔN mean	RMS
			Abs. T.	ϕ – Dir	λ – Dir		
Cairo Area	8	OSU81	35.1428	-2.91848	1.63958	-1.2708	0.0461
		Geoid84	74.7129	-4.93009	2.26587	-2.6199	0.0728
		OSU86	-15.0654	-1.84158	2.17221	-2.3204	0.0453
		OSU89*	10.0235	-2.59010	2.14536	-0.6184	0.0496
		OSU91	18.3825	-2.55728	1.82683	-1.2728	0.0450
		OSU91A	49.2075	-4.99205	3.16136	-1.8157	0.0826
		EGM96	27.470	-2.19560	1.20140	-0.9313	0.0345
Embaba Area	4	OSU81	-34.645	-1.25897	2.27436	-1.5665	0.0074
		Geoid84	20.4236	-3.47046	2.59762	-2.9220	0.0169
		OSU86	-68.4744	-0.29723	2.39459	2.7230	0.0040
		OSU89*	-60.6391	-1.49873	3.35458	-1.0777	0.0094
		OSU91	-57.3629	-1.37889	3.11486	-1.6747	0.0087
		OSU91A	-7.16298	-3.48236	3.51322	-2.3077	0.0177
		EGM96	-52.9137	-1.13893	2.75429	-1.2559	0.0074
Alexandria Area	5	OSU81	69.2117	-1.00489	-1.32928	-1.9329	0.0142
		Geoid84	86.2297	-3.44326	0.519237	-5.5940	0.0214
		OSU86	107.6830	-1.60524	-2.04964	-3.7552	0.0212
		OSU89	91.9204	-2.20634	-0.84906	-2.3086	0.0179
		OSU91	91.5402	-1.96665	-1.08940	-2.4119	0.0177
		OSU91A	-6.55189	0.66239	-0.58271	-3.3464	0.0072
		EGM96*	96.28630	-2.08528	-1.08803	-1.3240	0.0183
Port Said Area	4	OSU81	-170.1610	4.26804	1.08042	-1.8253	0.0079
		Geoid84	-70.5928	2.47653	-0.337775	-4.0154	0.0139
		OSU86	-84.2910	2.87267	-0.269813	-3.1311	0.0142
		OSU89*	-61.7410	2.06904	-0.135594	-1.3919	0.0103
		OSU91	-57.5184	2.14835	-0.367943	-2.1791	0.0134
		OSU91A	-126.1950	4.15072	-0.173232	-1.9435	0.0169
		EGM96	-76.1338	2.14858	0.232001	-1.4465	0.0069
Assuit Area	5	OSU81	3.08483	1.27223	-1.23778	0.8992	0.0428
		Geoid84	32.3161	-0.755294	-0.42924	-1.4232	0.0083
		OSU86	-20.5736	1.64212	-0.875882	-3.3301	0.0415
		OSU89*	-76.8220	0.681733	1.88666	0.1307	0.0263
		OSU91	-68.4226	0.718435	1.56593	-0.4251	0.0201
		OSU91A	-14.3019	-0.489592	0.849165	-1.2051	0.0242
		EGM96	-19.2192	0.441794	0.204903	-0.9161	0.0075
New Vally Area	6	OSU81	-116.3000	-1.56403	5.00136	-3.1539	0.0385
		Geoid84*	-151.5860	-2.05283	6.66471	-0.0112	0.0454
		OSU86	-118.0750	-1.92962	5.33087	-4.1631	0.0429
		OSU89	-241.8200	-0.247853	8.0863	-0.7739	0.0336
		OSU91	-229.309	-0.362497	7.75745	-1.2435	0.0333
		OSU91A	-186.785	-1.4549	7.29339	-0.7460	0.0394
		EGM96	-180.0160	-0.843718	6.56001	-0.8403	0.0336

To be continued.

Table (7), Continued

Zone	St. No.	Used GM	Polynomial Coefficients			ΔN mean	RMS
			Abs. T.	ϕ – Dir	λ - Dir		
Abo-Simbul Area	5	OSU81	256.837	-6.75455	-3.38589	-1.2861	0.0614
		Geoid84	243.378	-6.62044	-2.97126	1.3626	0.0633
		OSU86	232.455	-6.15362	-3.02615	-0.8546	0.0564
		OSU89	220.790	-5.55543	-3.02662	0.8459	0.0483
		OSU91*	230.457	-5.67509	-3.26612	0.2646	0.0478
		OSU91A	248.881	-6.55535	-3.25027	0.4991	0.0599
		EGM96	200.241	-5.19573	-2.66644	-0.2708	0.0467
Al Arish Area	6	OSU81	-219.707	2.96812	3.68114	-2.9393	0.0241
		Geoid84	-155.050	-0.528818	4.95825	-3.7309	0.0494
		OSU86	-244.683	2.75055	4.6074	-3.3377	0.0258
		OSU89	-218.496	2.01775	4.55332	-1.7504	0.0268
		OSU91	-219.323	2.1377	4.43329	-2.9117	0.0257
		OSU91A	-127.198	-0.084412	3.76431	-2.4644	0.0353
		EGM96*	-237.819	2.15937	4.99847	-1.6126	0.0291
El Tor Area	5	OSU81	-330.672	3.31992	7.13275	3.0096	0.1525
		Geoid84*	-225.760	3.12926	4.08974	0.1415	0.0814
		OSU86	-456.776	7.69186	7.22108	3.2097	0.1338
		OSU89	-392.813	6.37125	6.50180	5.7137	0.1229
		OSU91	-386.482	6.25049	6.38149	4.5903	0.1207
		OSU91A	-222.112	3.43145	3.82093	3.2688	0.0737
		EGM96	-317.243	5.17136	5.18409	3.0921	0.0976
Sant Katrin Area	4	OSU81	211.369	-4.81628	-1.99355	5.2951	0.0459
		Geoid84*	69.9337	-1.32171	-0.851133	3.0238	0.0132
		OSU86	121.433	-8.1865	3.56945	8.1877	0.0707
		OSU89	-35.3397	-4.91962	5.48931	10.5321	0.0431
		OSU91	-32.4677	-4.91952	5.36946	9.3242	0.0430
		OSU91A	110.145	-1.31269	-1.95446	5.9075	0.0157
		EGM96	-14.4099	-5.02962	4.84698	6.4251	0.0436
Ras El-Nakab Area	4	OSU81*	160.788	0.863632	-5.36262	-0.0804	0.0505
		Geoid84	119.495	0.860245	-4.14158	0.9749	0.0395
		OSU86	255.134	-0.375084	-7.04931	-1.0254	0.0725
		OSU89	439.378	-3.18023	-9.74319	6.5604	0.1322
		OSU91	444.085	-3.20234	-9.89863	5.2093	0.1339
		OSU91A	215.647	-0.134767	-6.0262	2.1664	0.0603
		EGM96	454.327	-3.51553	-10.009	2.3464	0.1397
Al Gora Area	5	OSU81	12.1847	0.140476	-0.55333	-2.3397	0.0119
		Geoid84	57.866	-2.84763	0.817148	-2.7371	0.0393
		OSU86	-49.9355	0.947735	0.530255	-2.3788	0.0106
		OSU89	10.3669	0.236603	-0.549061	-1.0243	0.0127
		OSU91	15.5849	0.292603	-0.788165	-2.2287	0.0161
		OSU91A	86.022	-2.39243	-0.395899	-1.8468	0.0221
		EGM96*	49.9501	-0.243935	-1.26984	-0.9823	0.0192

To be continued,

Table (7), Continued

Zone	St. No.	Used GM	Polynomial Coefficients			ΔN mean	RMS
			Abs. T.	ϕ – Dir	λ - Dir		
Sharm ElSheikh Area	5	OSU81	185.754	3.50344	-8.1216	4.4348	0.0943
		Geoid84*	107.095	3.66746	-6.06022	1.2649	0.0594
		OSU86	-136.115	11.0684	-4.88621	5.5019	0.1067
		OSU89	-90.8392	11.4287	-6.44377	7.2884	0.0950
		OSU91	-84.5217	11.3085	-6.56354	6.1236	0.0923
		OSU91A	151.836	4.43612	-7.92239	3.4630	0.0806
		EGM96	-0.118129	8.66534	-6.92337	4.1979	0.0610
Aswan Area	4	OSU81	611.012	-25.5764	0.065215	0.2911	0.4335
		Geoid84	534.574	-24.1886	1.46027	2.8959	0.4095
		OSU86*	609.789	-25.215	0.17480	-0.1496	0.4274
		OSU89	465.278	-22.694	2.4663	2.4318	0.3839
		OSU91	467.322	-22.8139	2.46642	1.6064	0.3859
		OSU91A	545.465	-23.4921	0.545572	0.4537	0.3979
		EGM96	484.429	-23.5344	2.46622	1.4438	0.3981
Marsa Matrouh Area	4	OSU81	3.00528	2.81155	-3.41598	-1.9198	0.0596
		Geoid84	270.629	-4.30329	-5.25337	-7.1843	0.0849
		OSU86	-5.42114	2.51465	-2.77063	-2.0777	0.0508
		OSU89	-95.0499	5.84518	-3.37402	-3.8059	0.0957
		OSU91	-91.4471	5.85156	-3.48566	-3.0427	0.0965
		OSU91A*	74.4419	2.4174	-5.55335	-1.0167	0.0774
		EGM96	-70.4923	5.12818	-3.36599	-1.4897	0.0865
Hurghada Area	4	OSU81	-99.6133	3.73845	-0.031459	0.9240	0.0786
		Geoid84	71.7815	-0.081796	-2.04401	0.4636	0.0321
		OSU86	-108.942	3.67212	0.276329	0.1967	0.0757
		OSU89	-88.2516	3.4617	-0.091405	2.7379	0.0732
		OSU91	-79.6342	3.25212	-0.208885	1.6884	0.0695
		OSU91A	60.0801	1.27641	-2.72913	2.5149	0.0580
		EGM96*	-128.951	3.46282	1.02971	-0.0329	0.0688
Luxor Area	4	OSU81	-107.989	2.00632	1.69098	-1.1805	0.0322
		Geoid84*	-23.3746	-0.511638	1.11614	-0.0052	0.0099
		OSU86	-8.62481	0.146964	0.066458	-2.6785	0.0074
		OSU89	-35.269	0.597951	0.634637	0.8373	0.0126
		OSU91	-32.1974	0.597947	0.514591	-0.0174	0.0119
		OSU91A	-57.7843	1.15079	0.860039	-0.1140	0.0187
		EGM96	-15.2519	-0.12188	0.514902	-1.5407	0.0081